

DSC 410/510 Multivariate Statistical Methods

Multidimensional Scaling

1

What is Multidimensional Scaling?

- MDS is a form of **Perceptual Mapping**
- Marketing: identify key dimensions underlying customer evaluations of products, services, companies, etc.
- Based on similarity or preference judgments by respondents on objects
 - ◆ no explicit use of attributes or features
- Transform these judgments into distances represented in multidimensional space (often two-dimensional scatterplots)

2

Why Use MDS?

- Why not just use Conjoint Analysis?
- Customer's **perceived (subjective)** dimensions (attributes) may not match firm's **objective** dimensions
 - ◆ objective: product features, color, etc...
 - ◆ perceived: quality, image, etc...
- Dimensions may interact to create unexpected evaluations
 - ◆ lack of additivity

3

Candy Bar Example 1

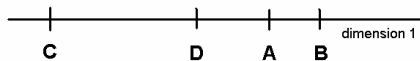
- Rank-order (1-15) similarities of each pair of candy bars:

candy bar	A	B	C	D	E	F
A	0	-	-	-	-	-
B	2	0	-	-	-	-
C	13	12	0	-	-	-
D	4	6	9	0	-	-
E	3	5	10	1	0	-
F	8	7	11	14	15	0

4

One-dimensional Solution

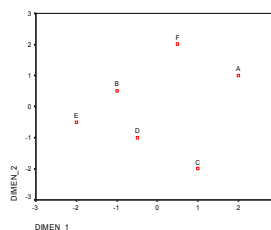
- Position object along a single dimension to preserve ranks
 - ◆ nearly possible with 4 candy bars
 - ◆ impossible with more



5

Two-dimensional Solution: Iteration 0

- Randomly place objects on a scatterplot:



6

Badness-of-Fit Criterion (Stress)

$$\sqrt{\frac{\sum (d_{ij} - \hat{d}_{ij})^2}{\sum (d_{ij} - \bar{d})^2}}$$

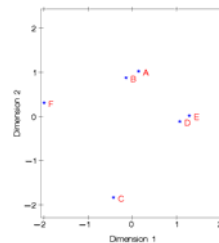
data distances map distances
average map distance

- Measure of how poorly MDS model fits
- Compare:
 - ◆ respondent (data) similarities, rescaled to represent distances
 - ◆ distances (on the perceptual map)

7

Two-dimensional Solution: Iterations 1-5

- Move objects to minimize Badness-of-Fit:



8

What are the Dimensions?

- Map can often suggest what they are
- e.g. d and e may be chocolate bars; a, b and c chocolate-covered; f fruit-based
 - ◆ i.e. dimension 1 measures “chocolateness”
- e.g. a and b may be nut-based; d, e and f some nuts; c no nuts
 - ◆ i.e. dimension 2 measures “nuttness”
- Need additional data to confirm this

9

MDS as a Clustering Technique

- Perceptual map can suggest clusters, so useful for segmentation
- But, it differs from cluster analysis:
 - ◆ analysis can be done at two levels:
 - ◆ individual (disaggregate)
 - ◆ aggregate
 - ◆ no *variables* measured on each object, just similarities between objects

10

MDS Objectives

- Exploratory technique to identify dimensions affecting consumer behavior
- To obtain comparative evaluations of objects where specific modes of comparison are unknown or undefined
- Objects must share a common basis of comparison
 - ◆ but no need for defined attributes

11

Data Considerations

- Include all relevant objects, but exclude irrelevant, inappropriate or noncomparable objects
- Objects are compared to one another (using similarities), e.g.
 - ◆ rank-order all pairs
 - ◆ assess similarity of each pair on a scale, say from 1-9 (more common)

12

Other Perceptual Map Techniques

- Preference-based perceptual maps use “good-bad” assessments
- MDS is a **decompositional** (attribute-free) technique, but **compositional** methods can be used to draw perceptual maps, e.g. correspondence analysis
- Compositional results are more easily interpreted, but restricted to specified attributes at the aggregate level and require increased data collection effort

13

How Many Objects?

- Guideline: more than 4 times as many objects as dimensions
 - ◆ 5 or more for 1 dimension
 - ◆ 9 or more for 2 dimensions
 - ◆ 13 or more for 3 dimensions
- But, $n(n-1)/2$ comparisons have to be made:
 - ◆ 36 for 9 objects
 - ◆ 78 for 13 objects!

14

Nonmetric or Metric Analysis?

- Similarities can be modeled as nonmetric (ordinal) or metric (interval or ratio)
- Nonmetric methods try to preserve ordered relationships in the input data
 - ◆ allows flexible relationship between similarities and map distances
- Metric methods try to preserve quantitative qualities of the input data (in addition to ordered relationships)

15

Data Collection

- (Direct) similarities of each pair
 - ◆ either rank-order
 - ◆ or rate on a scale from 1 (very similar) to 9 (not at all similar)
- Similarities derived from attribute data
 - ◆ e.g. based on distances between objects
 - ◆ aggregate analysis only
- Confusion data or subjective clustering
 - ◆ sort objects into similar piles
 - ◆ aggregate analysis only

16

MDS Assumptions

- No statistical assumptions!
- However, MDS techniques rely on psychological assumptions about perception
 - ◆ underlying dimensions vary from person to person
 - ◆ dimensions that match vary in importance
 - ◆ perceptions tend to vary over time
- MDS attempts to identify common underlying dimensions and relationships

17

Finding a MDS Solution

- Variety of algorithms and software available
- SAS uses Proc MDS or the multidimensional scaling method in the Market Research application
 - ◆ Measurement level can be metric or nonmetric
 - ◆ Analysis can be done at the individual level (disaggregate analysis) and/or the aggregate level

18

Aggregate Analyses

- Produce individual maps and average them to produce a group map – not recommended
- Average the similarity matrices and produce a group map from the average matrix – better but still not optimal
- Derive a group map that simultaneously minimizes individual stresses (SAS does something along these lines)
 - ◆ “individual differences analysis” goes a step further, allowing individuals to differ from group map by weighting dimensions differently
 - ◆ e.g. candy bar example 2

19

SAS Output

- Two-dimensional maps:
 - ◆ coordinates (shows objects)
 - ◆ coefficients (shows subjects if an individual differences analysis has been done)
- Fit statistics (stress and correlation measures)
 - ◆ for each respondent
 - ◆ Aggregate
- Configuration table displays numbers used to construct maps

20

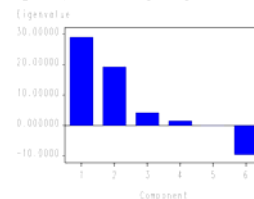
Is the Group Homogenous?

- Coefficients map (containing subjects) can indicate clusters and/or unusual subjects
- If there is a lot of separation between clusters and/or individual points, group homogeneity might be doubtful
 - ◆ then run separate analyses
 - ◆ e.g. candy bar example 3
- Much of what the book says on this subject does not apply to SAS analyses

21

Selecting Dimensionality

- Use a scree plot (retain “large eigenvalues”):



- Look for a reasonable fit correlation (close to +1)
- 3+ dimensions hard to interpret, 1 seldom works well, so usually just pick 2!

22

Interpretation and Validation

- Attempt to describe the perceptual dimensions and their correspondence to attributes
 - ◆ subjective: use your own judgment based on what you see in the map
 - ◆ objective: more formalized methods are available but require specialized software
- Validate results using:
 - ◆ holdout sample comparison
 - ◆ alternative method comparison

23

Examples

- Local restaurant example in class
 - ◆ **restaurant.xls** (blank Excel spreadsheet to be filled in for assignment 13)
 - ◆ **restaurantall.xls** (Excel spreadsheet containing class data)
- Illustration of MDS from the text-book (p555-559)
 - ◆ **hatcomds.xls** (Excel spreadsheet with data)
 - ◆ “Illustration of MDS from p555-559” handout

24