

DSC 410/510 Multivariate Statistical Methods

Correspondence Analysis

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What is Correspondence Analysis

- A **compositional** perceptual mapping technique
- Derives associations between:
 - ◆ objects
 - ◆ attributes
 - ◆ objects and attributes
- How well do different categories of objects/attributes correspond to each other?

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Product Sales Example

- Contingency table: frequencies of occurrence in a cross-tabulation of product (objects A, B, and C) and age (attributes 18-35, 36-55, 56+):

Age	A	B	C	
Young	20	20	20	60
Middle	40	10	40	90
Senior	20	10	40	70
	80	40	100	

If you have a fixed budget to spend promoting product A, who would you target? What about product B? And C?

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Identify Patterns in Table

- Deciding who to target should take into account market share *within age category*
- Identifying product each age group is most closely associated with should consider market share *within product category*
- i.e. which product/age combinations have higher sales than might be expected if there was no relationship between product and age

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Expected Sales

- If there was no relationship between product category and age category, sales would be proportional to row/column totals:

Age	A	B	C	
Young	21.82	10.91	27.27	60
Middle	32.73	16.36	40.91	90
Senior	25.45	12.73	31.82	70
	80	40	100	

For example, expected sales of product A to 18-35 year olds is $60/(60+90+70) \times 80 = 60/220 \times 80 = 21.82$

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Actual – Expected Sales

- Negative numbers indicate fewer actual sales than expected, positive numbers indicate higher actual sales than expected:

Age	A	B	C	
Young	-1.82	9.09	-7.27	0
Middle	7.27	-6.36	-0.91	0
Senior	-5.45	-2.73	8.18	0
	0	0	0	

But, we need to take into account the fact that expected sales were only 10.91 for Young/B, but 31.82 for Senior/C

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Standardize: Chi-square Values

- Square the difference and then divide by expected sales to get the chi-square values
- Sign of difference gives similarities:

Age	A	B	C
Young	-0.15	7.58	-1.94
Middle	1.62	-2.47	-0.02
Senior	-1.17	-0.58	2.10

For example, χ^2 for Young/A is $(1.82)^2/21.82 = 0.15$ while similarity value is -0.15 since sign of difference is "minus"

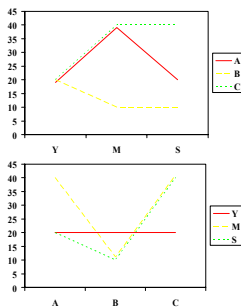
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Correspondences

- **Objects** correspond to one another if their column profiles are similar
 - ◆ e.g. see next slide
- **Attributes** correspond to one another if their row profiles are similar
 - ◆ e.g. see next slide
- **Objects** correspond to **attributes** (and vice versa) if their chi-square similarity values are high (and positive)
 - ◆ e.g. in the perceptual map, B & Y should be close together, as should A & M, and C & S

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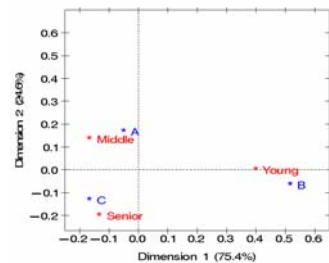
Product Sales Example



- Column profiles: A and C are most similar
 - ◆ so in the perceptual map, A and C should be closer to each other (than they are to B)
- Row profiles: M and S are most similar
 - ◆ so in the perceptual map, M and S should be closer to each other (than they are to Y)

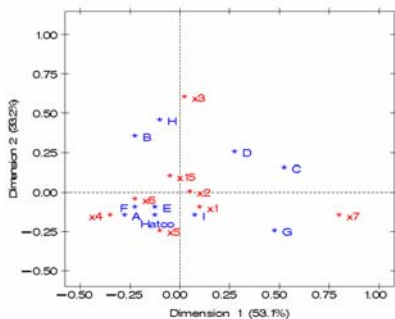
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Perceptual Map from SAS



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Hatco Example (p565-570)



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